MADPH-00-1177
UH-511-964-00
VAND-TH-00-5
AMES-HET 00-04
May 2000

CPT-odd Resonances in Neutrino Oscillations

V. Barger[1], S. Pakvasa[2], T.J. Weiler[3], and K. Whisnant[4]

[1]*Department of Physics, University of Wisconsin, Madison, WI 53706, USA*
[2]*Department of Physics and Astronomy, University of Hawaii, Honolulu, HI 96822, USA*
[3]*Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235, USA*
[4]*Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA*

Abstract

We consider the consequences for future neutrino factory experiments of small CPT-odd interactions in neutrino oscillations. The $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ survival probabilities at a baseline $L = 732$ km can test for CPT-odd contributions at orders of magnitude better sensitivity than present limits. Interference between the CPT-violating interaction and CPT-even mass terms in the Lagrangian can lead to a resonant enhancement of the oscillation amplitude. For oscillations in matter, a simultaneous enhancement of both neutrino and antineutrino oscillation amplitudes is possible.

I. INTRODUCTION

The discrete symmetries C, P, and T have fundamental importance in elementary particle theory. Violations of C, P, CP and T by the weak interactions have all been observed [1]. CPT invariance is a basic property of local quantum field theory [2] and no evidence of deviations from CPT invariance have been found so far. The most stringent limit on CPT violation is obtained from the difference of the K^0 and $\bar{K}^0$ masses [3],

$$m_K - m_{\bar{K}} < 0.44 \times 10^{-18}\ \mathrm{GeV}\,. \tag{1}$$

In string theory, the CPT invariance may not be manifest due to the extended nature of strings [4,5]. Mechanisms by which string theories could spontaneously break CPT have been formulated [4,5]. The search for CPT violation is thus of considerable theoretical interest as a means of searching for purely string effects. Neutrino oscillations have been considered as phenomena that could probe CPT non-conservation [6]. With growing interest in the construction of neutrino factories to make high-precision measurements of neutrino mass-squared differences and of the CP violating phase in the neutrino sector [7], it is appropriate to undertake a more extensive study of the ability to measure CPT-violating effects in neutrino oscillations. We find that a comparison of $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ oscillation probabilities at neutrino factories would give precision tests of CPT. A significant result of our study, as reported below, is that CPT violating resonance effects can occur that can magnify small CPT violation into a measureable oscillation amplitude.

II. BASIC FORMALISM

Consequences of CP, T and CPT violation for neutrino oscillations have been written down before [8]. We summarize them briefly for the $\nu_\alpha \to \nu_\beta$ flavor oscillation probabilities $P_{\alpha\beta}$ at a distance L from the source. If

$$P_{\alpha\beta}(L) \neq P_{\bar{\alpha}\bar{\beta}}(L)\,, \qquad \beta \neq \alpha\,, \tag{2}$$

then CP is not conserved. If

$$P_{\alpha\beta}(L) \neq P_{\beta\alpha}(L)\,, \qquad \beta \neq \alpha\,, \tag{3}$$

then T-invariance is violated. If

$$P_{\alpha\beta}(L) \neq P_{\bar{\beta}\bar{\alpha}}(L)\,, \qquad \beta \neq \alpha\,, \tag{4}$$

or

$$P_{\alpha\alpha}(L) \neq P_{\bar{\alpha}\bar{\alpha}}(L)\,, \tag{5}$$

then CPT is violated. When neutrinos propagate in matter, matter effects give rise to apparent CP *and* CPT violation even if the mass matrix is CP conserving.

The CPT violating terms can be Lorentz-invariance violating (LV) or Lorentz invariant. The Lorentz-invariance violating, CPT violating case has been discussed by Coleman and Glashow [6] and by Colladay and Kostelecky [9]. We will consider this first.

The effective LV CPT violating interaction for neutrinos is of the form

$$\bar{\nu}_L^\alpha b_\mu^{\alpha\beta} \gamma_\mu \nu_L^\beta \,, \tag{6}$$

where α and β are flavor indices. We assume rotational invariance in the "preferred" frame, in which the cosmic microwave background radiation is isotropic (following Coleman and Glashow [6]). [1] The energies of ultra-relativistic neutrinos with definite momentum p are eigenvalues of the matrix

$$m^2/2p + b_0 \,, \tag{7}$$

where b_0 is a hermitian matrix, hereafter labeled b.

In the two-flavor case the neutrino phases may be chosen such that b is real, in which case the interaction in Eq. (6) is CPT-odd. The survival probabilities for flavors α and $\bar{\alpha}$ produced at $t = 0$ are given by [6]

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\Theta \sin^2(\Delta L/4) \,, \tag{8}$$

and

$$P_{\bar{\alpha}\bar{\alpha}}(L) = 1 - \sin^2 2\bar{\Theta} \sin^2(\bar{\Delta} L/4) \,, \tag{9}$$

where

$$\Delta \sin 2\Theta = \left| (\delta m^2/E) \sin 2\theta_m + 2\delta b e^{i\eta} \sin 2\theta_b \right| \,, \tag{10}$$

$$\Delta \cos 2\Theta = (\delta m^2/E) \cos 2\theta_m + 2\delta b \cos 2\theta_b \,. \tag{11}$$

$\bar{\Delta}$ and $\bar{\Theta}$ are defined by similar equations with $\delta b \to -\delta b$. Here θ_m and θ_b define the rotation angles that diagonalize m^2 and b, respectively, $\delta m^2 = m_2^2 - m_1^2$ and $\delta b = b_2 - b_1$, where m_i^2 and b_i are the respective eigenvalues. We use the convention that $\cos 2\theta_m$ and $\cos 2\theta_b$ are positive and that δm^2 and δb can have either sign. The phase η in Eq. (10) is the difference of the phases in the unitary matrices that diagonalize δm^2 and δb; only one of these two phases can be absorbed by a redefinition of the neutrino states.

Observable CPT-violation in the two-flavor case is a consequence of the interference of the δm^2 terms (which are CPT-even) and the LV terms in Eq. (6) (which are CPT-odd); if $\delta m^2 = 0$ or $\delta b = 0$, then there is no observable CPT-violating effect in neutrino oscillations. If $\delta m^2/E \gg 2\delta b$ then $\Theta \simeq \theta_m$ and $\Delta \simeq \delta m^2/E$, whereas if $\delta m^2/E \ll 2\delta b$ then $\Theta \simeq \theta_b$ and $\Delta \simeq 2\delta b$. Hence the effective mixing angle and oscillation wavelength can vary dramatically with E for appropriate values of δb.

We note that a CPT-odd resonance for neutrinos ($\sin^2 2\Theta = 1$) occurs whenever $\cos 2\Theta = 0$ or

$$(\delta m^2/E) \cos 2\theta_m + 2\delta b \cos 2\theta_b = 0 \,; \tag{12}$$

similar to the resonance due to matter effects [11,12]. The condition for antineutrinos is the same except δb is replaced by $-\delta b$. The resonance occurs for neutrinos if δm^2 and δb have

---

[1] An experimental limit on b_3^{ee} of 10^{-29} GeV has been obtained [10] in studies of torques on a spin polarized torsion pendulum. This translates into a bound on the ee element of the matrix b_0 of 5×10^{-25} GeV; if $SU(2)_L$ symmetry holds, a similar bound is implied on the $\nu_e\nu_e$ element of b_0, but there are no similar bounds on other $\nu\nu$ elements of b_0.

the opposite sign, and for antineutrinos if they have the same sign. A resonance can occur even when θ_m and θ_b are both small, and for all values of η; if $\theta_m = \theta_b$, a resonance can occur only if $\eta \neq 0$.

If one of ν_α or ν_β is ν_e, then the neutrino propagation is modified in the presence of matter. Then Eq. (11) becomes

$$\Delta \cos 2\Theta = (\delta m^2/E) \cos 2\theta_m + 2\delta b \cos 2\theta_b - 2\sqrt{2} G_F N_e \,, \tag{13}$$

for neutrinos, where N_e is the number density of electrons in matter. For antineutrinos, $\delta b \to -\delta b$ and $N_e \to -N_e$ in Eq. (13).

III. EXAMPLES OF CPT-VIOLATION AND CPT-ODD RESONANCES

Hereafter, for simplicity, we assume that m^2 and b are diagonalized by the same angle θ, i.e., $\theta_m = \theta_b \equiv \theta$.

A. $\eta = 0$

For $\eta = 0$ we have

$$\Theta = \theta \,, \tag{14}$$

$$\Delta = (\delta m^2/E) + 2\delta b \,. \tag{15}$$

In this $\theta_m = \theta_b$, $\eta = 0$ case a resonance is not possible. The oscillation probabilities become

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\theta \sin^2 \left\{ \left(\frac{\delta m^2}{4E} + \frac{\delta b}{2} \right) L \right\} \,, \tag{16}$$

$$P_{\bar\alpha\bar\alpha}(L) = 1 - \sin^2 2\theta \sin^2 \left\{ \left(\frac{\delta m^2}{4E} - \frac{\delta b}{2} \right) L \right\} \,. \tag{17}$$

For fixed E, the δb terms act as a phase shift in the oscillation argument; for fixed L, the δb terms act as a modification of the oscillation wavelength.

An approximate direct limit on δb when $\alpha = \mu$ can be obtained by noting that in atmospheric neutrino data the flux of downward going ν_μ is not depleted whereas that of upward going ν_μ is [13]. Hence, the oscillation arguments in Eqs. (16) and (17) cannot have fully developed for downward neutrinos. Taking $|\delta b L/2| < \pi/2$ with $L \sim 20$ km for downward events leads to the upper bound $|\delta b| < 3 \times 10^{-20}$ GeV; upward going events could in principle test $|\delta b|$ as low as 5×10^{-23} GeV. Since the CPT-odd oscillation argument depends on L and the ordinary oscillation argument on L/E, improved direct limits could be obtained by a dedicated study of the energy and zenith angle dependence of the atmospheric neutrino data.

The difference between $P_{\alpha\alpha}$ and $P_{\bar\alpha\bar\alpha}$

$$P_{\alpha\alpha}(L) - P_{\bar\alpha\bar\alpha}(L) = -2 \sin^2 2\theta \sin\left(\frac{\delta m^2 L}{2E} \right) \sin(\delta b L) \,, \tag{18}$$

can be used to test for CPT-violation. In a neutrino factory, the ratio of $\bar{\nu}_\mu \to \bar{\nu}_\mu$ to $\nu_\mu \to \nu_\mu$ events will differ from the standard model (or any local quantum field theory model) value if CPT is violated. Figure 1 shows the event ratios $N(\bar{\nu}_\mu \to \bar{\nu}_\mu)/N(\nu_\mu \to \nu_\mu)$ versus δb for a neutrino factory with 10^{19} stored muons and a 10 kt detector at several values of stored muon energy, assuming $\delta m^2 = 3.5 \times 10^{-3}$ eV^2 and $\sin^2 2\theta = 1.0$, as indicated by the atmospheric neutrino data [13]. The error bars in Fig. 1 are representative statistical uncertainties. The node near $\delta b = 8 \times 10^{-22}$ GeV is a consequence of the fact that $P_{\alpha\alpha} = P_{\bar{\alpha}\bar{\alpha}}$, independent of E, whenever $\delta b L = n\pi$, where n is any integer; the node in Fig. 1 is for $n = 1$. A 3σ CPT violation effect is possible in such an experiment for δb as low as 3×10^{-23} GeV for stored muon energies of 20 GeV. Although matter effects also induce an apparent CPT-violating effect, the dominant oscillation here is $\nu_\mu \to \nu_\tau$, which has no matter corrections in the two-neutrino limit; in any event, the matter effect is in general small for distances much shorter than the Earth's radius.

We have also checked the observability of CPT violation at other distances, assuming the same neutrino factory parameters used above. For $L = 250$ km, the $\delta b L$ oscillation argument in Eq. (18) has not fully developed and the ratio of $\bar{\nu}$ to ν events is still relatively close to the standard model value. For $L = 2900$ km, a δb as low as 10^{-23} GeV may be observable at the 3σ level. However, longer distances may also have matter effects that simulate CPT violation.

B. $\eta = \pi/2$

For $\eta = \pi/2$ we have

$$P_{\alpha\alpha} = 1 - \sin^2 2\Theta \sin^2 \left\{\Delta L/4\right\} , \tag{19}$$

$$P_{\bar{\alpha}\bar{\alpha}} = 1 - \sin^2 2\bar{\Theta} \sin^2 \left\{\bar{\Delta} L/4\right\} , \tag{20}$$

where

$$\tan 2\Theta = \frac{\sqrt{(\delta m^2/E)^2 + (2\delta b)^2}}{(\delta m^2/E) + 2\delta b} \tan 2\theta \, , \tag{21}$$

$$\Delta^2 = \left[(\delta m^2/E) + 2\delta b\right]^2 - 4(\delta m^2/E)\delta b \sin^2 2\theta \, , \tag{22}$$

and $\bar{\Theta}$ and $\bar{\Delta}$ are defined similarly with $\delta b \to -\delta b$. In this case the resonance condition for neutrinos is $\delta m^2/E + 2\delta b = 0$. Figure 2 shows the effective oscillation amplitude $\sin^2 2\Theta$ and oscillation argument Δ versus δb with $\delta m^2 = 3.5 \times 10^{-3}$ eV^2 and $\sin^2 2\theta = 0.1$ (which may be appropriate for $\nu_e \to \nu_e$ oscillations) for several values of neutrino energy. Although the above example assumed $\eta = \pi/2$, such a resonance can occur in this $\theta_b = \theta_m$ example for any value of η in the open interval $(0, 2\pi)$.

C. CPT-odd term with matter

In the presence of matter, the effective ν_e oscillation amplitude and argument are defined by Eqs. (10) and (13). Again assuming $\theta_b = \theta_m \equiv \theta$ and $\eta = 0$, we have

$$\tan 2\Theta = \frac{[(\delta m^2/E) + 2\delta b]\sin 2\theta}{[(\delta m^2/E) + 2\delta b]\cos 2\theta - 2\sqrt{2}G_F N_e}\,, \tag{23}$$

$$\Delta^2 = \left\{\left[(\delta m^2/E) + 2\delta b\right]\cos 2\theta - 2\sqrt{2}G_F N_e\right\}^2 + \left[(\delta m^2/E) + 2\delta b\right]^2 \sin^2 2\theta\,, \tag{24}$$

for neutrinos, with $\delta b \to -\delta b$ and $N_e \to -N_e$ for antineutrinos. Thus a resonance ($\sin^2 2\Theta = 1$) occurs for neutrinos when $[(\delta m^2/E) + 2\delta b]\cos 2\theta = 2\sqrt{2}G_F N_e$, and for antineutrinos when $[(\delta m^2/E) - 2\delta b]\cos 2\theta = -2\sqrt{2}G_F N_e$. A resonance can occur simultaneously for neutrinos *and* antineutrinos only in the limit when $\delta m^2/E \ll 2\delta b$ and the CPT-odd effects dominate. However, it is possible to have an effective oscillation amplitude that is significantly enhanced for both neutrinos and antineutrinos even when $\delta m^2/E$ is not small compared to $2\delta b$. For $N_e = 1.67 N_A/\mathrm{cm}^3$ (the electron density appropriate for the upper mantle of the Earth) and vacuum amplitude $\sin^2 2\theta = 0.1$, the effective oscillation amplitudes $\sin^2 2\Theta$ for $\nu_e \to \nu_e$ and $\sin^2 2\bar{\Theta}$ for $\bar{\nu}_e \to \bar{\nu}_e$ can both be greater than 0.5 when δb and δm^2 satisfy both $.0002\ \mathrm{eV}^2/\mathrm{GeV} < 2\delta b + (\delta m^2/E) < .0004\ \mathrm{eV}^2/\mathrm{GeV}$ and $.0002\ \mathrm{eV}^2/\mathrm{GeV} < 2\delta b - (\delta m^2/E) < .0004\ \mathrm{eV}^2/\mathrm{GeV}$. These conditions are satisfied when $\delta b \simeq 1\text{–}2 \times 10^{-22}$ GeV and with $|\delta m^2/E|$ as large as $10^{-4}\ \mathrm{eV}^2/\mathrm{GeV}$. Assuming $\delta m^2 \simeq 3.5 \times 10^{-3}\ \mathrm{eV}^2$, such enhancements in $\nu_e \to \nu_e$ and $\bar{\nu}_e \to \bar{\nu}_e$ are possible for $E > 35$ GeV, provided that $\delta b > 0$. Although here we have considered the case $\eta = 0$, similar enhancements are possible for any value of η since they rely on the denominator of Eq. (23) being small, which is independent of η.

IV. LORENTZ-INVARIANT CASE

We can simulate a possible Lorentz-invariant CPT-odd effective interaction by allowing the mass matrix for $\bar{\nu}$'s to be different from the one for ν's. If we assume, for simplicity, that the CPT-violating effects are more important in δm^2 than in mixing, then there is only one CPT-odd parameter, namely $\delta m^2 - \delta \bar{m}^2 = \epsilon$, and the oscillation probabilities are

$$P_{\alpha\alpha} = 1 - \sin^2 2\theta \sin^2[\delta m^2 L/(4E)]\,, \tag{25}$$

$$P_{\bar{\alpha}\bar{\alpha}} = 1 - \sin^2 2\theta \sin^2[(\delta m^2 - \epsilon)L/(4E)]\,. \tag{26}$$

From the lack of large disappearance of downward going atmospheric muons, we obtain an approximate upper bound of $|\epsilon| < 0.1\ \mathrm{eV}^2$ when $\alpha = \mu$. A fit to the total number of muon and antimuon events in the SuperK atmospheric neutrino data sample would greatly improve this bound.

V. SUMMARY

We have shown that small CPT-odd interactions of neutrinos can have measureable consequences in neutrino oscillations. Resonant enhancements of the oscillation amplitude for either neutrinos or antineutrinos (but not both) are possible if the unitary matrices which diagonalize the neutrino mass term and the CPT-odd term are not the same. A resonance can occur for any relative phase between the CPT-even mass term and the CPT-odd interaction, but if the rotation angles in the two sectors are the same, a resonance

is possible only if the relative phase is not zero. In matter, significant enhancements are possible for both neutrinos and antineutrinos. Measurement of $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ oscillation probabilities in neutrino factories can place stringent limits on the CPT-odd interaction.

ACKNOWLEDGMENTS

The authors would like to thank J. Learned, J. Lykken and X. Tata for useful discussions. This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grants No. DE-FG02-94ER40817, No. DE-FG05-85ER40226, and No. DE-FG02-95ER40896, and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

REFERENCES

[1] See e.g., I.I. Bigi, and A.I. Sanda, *CP Violation* (Cambridge Univ. Press, 2000); G.C. Branco, L. Lavoura, and J.P. Silva, *CP Violation* (Clarendon Press, Oxford, 1999); and references therein.

[2] See e.g., S. Weinberg, *The Quantum Theory of Fields* (Cambridge Univ. Press, 1996) p. 244, and references therein.

[3] Particle Data Group, C. Caso et al., Review of Particle Physics, Eur. J. Phys. C **3**, 1 (1998).

[4] V.A. Kostelecky and R. Potting, Nucl. Phys. B **359**, 545 (1991); Phys. Rev. D **51**, 3923 (1995).

[5] J. Ellis, J. L. Lopez, N. E. Mavromatos, and D. V. Nanopoulos, Phys. Rev. D **53**, 3846 (1996).

[6] S. Coleman and S.L. Glashow, Phys. Rev. D **59**, 116008 (1999).

[7] V. Barger, S. Geer, R. Raja, and K. Whisnant, hep-ph/0003184 (2000); A. De Rujula, M.B. Gavela, and P. Hernandez, Nucl. Phys. B **547**, 21 (1999); M. Campanelli, A. Bueno, and A. Rubbia, hep-ph/9905240; A. Cervera, A. Donini, M.B. Gavela, J.J. Gomez Cadenas, P. Hernandez, O. Mena, and S. Rigolin, hep-ph/0002108; I. Mocioiu and R. Shrock, hep-ph/9910554, hep-ph/0002149; M. Freund, M. Lindner, S.T. Petcov, and A. Romanino, hep-ph/9912457.

[8] N. Cabibbo, Phys. Lett. **B72**, 333 (1978); V. Barger, K. Whisnant, R.J.N. Phillips, Phys. Rev. Lett. **45**, 2084 (1980); S. Pakvasa, in *Proc. of the XXth International Conference on High Energy Physics*, ed. by L. Durand and L.G. Pondrom, AIP Conf. Proc. No. 68 (AIP, New York, 1981), Vol. 2, p. 1164.

[9] D. Colladay and V.A. Kostelecky, Phys. Rev. D **55**, 6760 (1997).

[10] E.G. Adelberger *et al.*, unpublished, as quoted by N. Russell, R. Blum, and V.A. Kostelecky, Bull. Am. Phys. Soc. **45**, No. 2, 140 (2000).

[11] V. Barger, K. Whisnant, S. Pakvasa, and R.J.N. Phillips, Phys. Rev. D **22**, 2718 (1980).

[12] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S.P. Mikheyev and A. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1986)].

[13] Kamiokande collaboration, K.S. Hirata *et al.*, Phys. Lett. **B280**, 146 (1992); Y. Fukuda *et al.*, Phys. Lett. **B335**, 237 (1994); IMB collaboration, R. Becker-Szendy *et al.*, Nucl. Phys. Proc. Suppl. **38 B**, 331 (1995); Soudan-2 collaboration, W.W.M. Allison *et al.*, Phys. Lett. **B391**, 491 (1997); MACRO collaboration, M. Ambrosio *et al.*, Phys. Lett. **B434**, 451 (1998); Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); Phys. Rev. Lett. **82**, 2644 (1999).

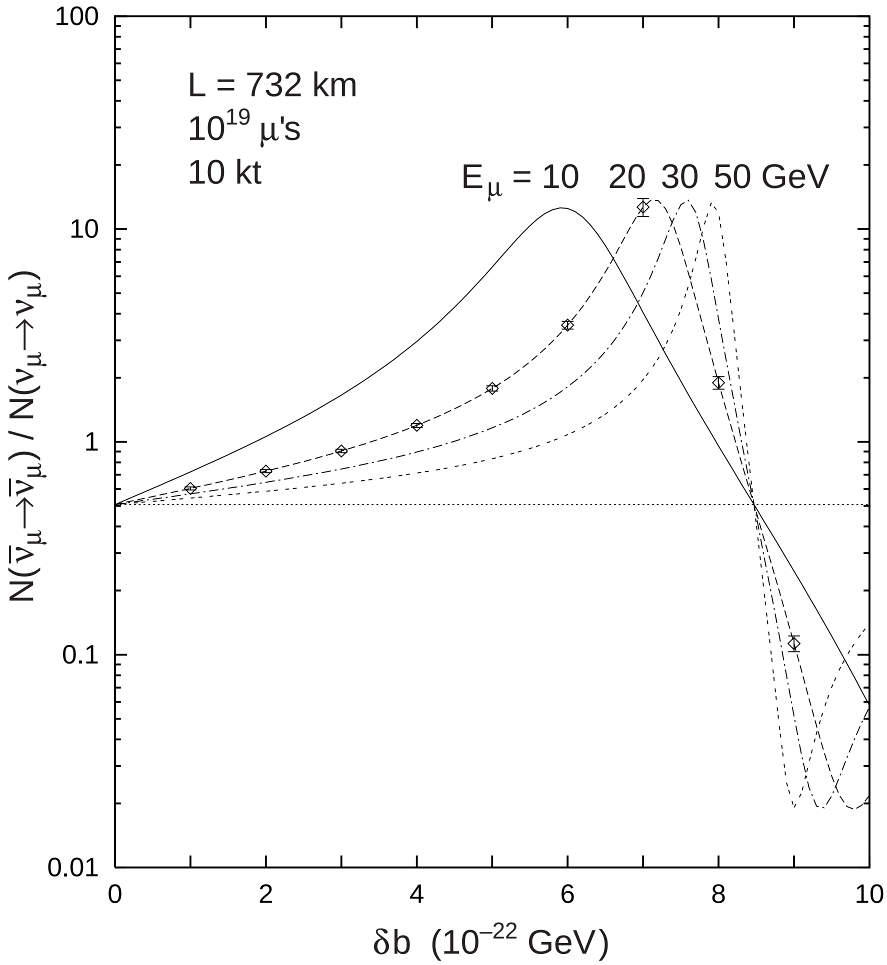

FIG. 1. The ratio of $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ to $\nu_\mu \rightarrow \nu_\mu$ event rates in a 10 kt detector for a neutrino factory with 10^{19} stored muon with energies $E_\mu = 10$, 20, 30, 50 GeV for baseline $L = 732$ km versus the CPT-odd parameter δb with $\theta_m = \theta_b \equiv \theta$ and phase $\eta = 0$. The neutrino mass and mixing parameters are $\delta m^2 = 3.5 \times 10^{-3}$ eV^2 and $\sin^2 2\theta = 1.0$. The dotted line indicates the result for $\delta b = 0$, which is given by the ratio of the $\bar{\nu}$ and ν charge-current cross sections. The error bars are representative statistical uncertainties.

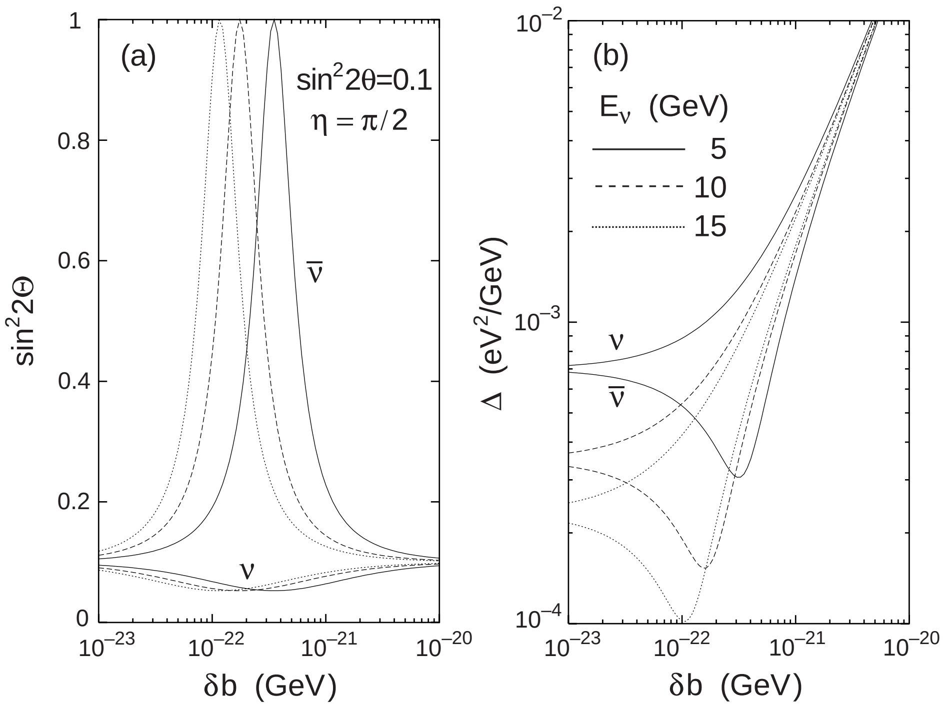

FIG. 2. Resonance effects in $\nu \to \nu$ and $\bar{\nu} \to \bar{\nu}$ oscillations shown versus CPT-odd parameter δb for various values of neutrino energy E with $\delta m^2 = 3.5 \times 10^{-3}\,\mathrm{eV}^2$, $\sin^2 2\theta = 0.1$ and phase $\eta = \pi/2$: (a) oscillation amplitude $\sin^2 2\Theta$ in Eq. (21) and (b) oscillation argument Δ in Eq. (22).